Power-Controlled Cognitive Radio Spectrum Allocation with Chemical Reaction Optimization

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***Abstract*—Cognitive radio is a promising technology for increasing the system capacity by using the radio spectrum more effectively. It has been widely studied recently and one important problem in this new paradigm is the allocation of radio spectrum to secondary users effectively in the presence of primary users. We call it the cognitive radio spectrum allocation problem (CRSAP) in this paper. In the conventional problem formulation, a secondary user can be either on or off and its interference range becomes maximum or zero, respectively. We first develop a solution to CRSAP based on the newly proposed chemical reaction-inspired metaheuristic called Chemical Reaction Optimization (CRO). We study different utility functions, accounting for utilization and fairness, with the consideration of the hardware constraint, and compare the performance of our proposed CRO-based algorithm with existing ones. Simulation results show that the CRO-based algorithm always outperforms the others dramatically. Next, by allowing adjustable transmission power, we propose power-controlled CRSAP (PC-CRSAP), a new formulation to the problem with the consideration of spatial diversity. We design a two-phase algorithm to solve PC-CRSAP, and again simulation results show excellent performance.**

***Index Terms*—Cognitive radio, channel allocation, chemical reaction optimization, evolutionary algorithm.**

I. Introduction

TO prevent interference of wireless signals, the frequency spectrum is divided into multiple bands for different purposes, and regulated by government agencies. Some spectrum bands are licensed and limited to the use of authorized users (primary users) while some (i.e. unlicensed bands) can be used without restriction. Due to underutilization of the licensed bands and overcrowding of the unlicensed bands, the capacity can be dramatically increased if the unlicensed users (secondary users) are allowed to use the licensed bands. However, primary users have priority in using their respective licensed bands and secondary users can only operate in these bands provided their activities do not affect the primary users. This paradigm, firstly proposed in [1], is known as cognitive radio opportunistic spectrum access.

One important problem in this new paradigm is the allocation of radio spectrum to secondary users effectively in the presence of primary users. We call it the cognitive radio spectrum allocation problem (CRSAP) in this paper. This problem can be solved by a centralized approach or by a distributed strategy. The former refers to the situation in which a central authority (e.g., a spectrum policy server) possesses all necessary primary and secondary user information in a given geographical area and assigns available spectrum segments to the secondary users [2], [3], [4]. This is particularly useful for infrastructure-based networks with static environmental and user conditions in a certain period of time. In the latter case, secondary users detect available channels themselves and negotiate channel acquisition with their neighbors according to local information [2], [5]. This favors decentralized ad hoc networks where centralized authorities are unavailable. In this work, we focus on centralized approaches.

CRSAP, a non-convex optimization problem, is proved to be *NP*-hard [2]. Since evolutionary computing techniques (e.g. Genetic Algorithm [6], Ant Colony Optimization [7], Particle Swarm Optimization (PSO) [8], and Chemical Reaction Optimization (CRO) [9]) have been successfully applied to these non-convex problems to give near-optimal results, we try to develop a CRO-based centralized algorithm to solve CRSAP. CRO is a (variable) population-based general-purpose optimization metaheuristic and has been successfully applied to many practical problems, e.g. channel assignment in wireless mesh networks [9], task scheduling in grid computing [10], population transition in peer-to-peer live streaming [11], and network coding optimization problem [12]. The state-of-the-art of CRO can be found in [13], [14]. CRO mimics the interactions of molecules driving towards the minimum state of free energy (i.e. the most stable state). The manipulated agents are molecules, each of which has a molecular structure, potential energy (*PE*), kinetic energy (*KE*), and some other optional attributes. The molecular structure and *PE* correspond to a solution of a given problem and its objective function value, respectively. *KE* represents the tolerance of a molecule getting a worse solution than the existing one, thus allowing CRO to escape from local optimum solutions. Imagine that we have a set of molecules in a closed container. They move and collide either on the walls of the container (uni-molecular collisions) or with each other (inter-molecular collisions). Each collision results in one of the four types of elementary reactions. They have different characteristics and extents of change to the solutions. With the conservation of energy, the solutions change from high to low energy states and we output the molecular structure with the lowest found *PE* as the best solution.

The conventional formulation of CRSAP assumes that the interference is either on or off, with the interference range either maximum or zero. It has been thoroughly studied in [2], [3], [4]. [2] employs a heuristic graph coloring approach to solve the problem. In [3], the authors utilize evolutionary approaches, including Canonical Genetic Algorithm (CGA) [6], Quantum Genetic Algorithm (QGA) [15], and PSO, and

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it is shown that these approaches are superior to the graph coloring method. [4] demonstrates that CRO has even better performance than those used in [3].

Based on the same set of system information, it is possible to improve the system utility via spatial diversity by allowing users to adjust their transmission powers, thus changing the transmission and interference ranges. We give the new formulation in light of spatial diversity and provide a way to modify the solution produced for CRSAP to a solution for the new formulation with improved utility. The contributions of this paper are summarized as follows:

- We apply a novel chemical reaction-inspired algorithm CRO to tackle CRSAP;
- We show that the solution of CRSAP can be improved with spatial diversity;
- We propose a new formulation of the problem to achieve spatial diversity by controlling the transmission powers of the secondary users;
- We propose a simple but powerful two-stage heuristic method, which can improve any existing solution to the original CRSAP, to solve the new problem;
- We demonstrate that CRO outperforms other existing methods substantially for the conventional formulation of the problem and our heuristic can improve the inferior solutions of CRSAP very effectively.

The rest of this paper is organized as follows. We give the conventional formulation in Section II. In Section III, we analyze a system with two secondary users and show that it is possible to enhance the system utility by spatial diversity. We propose the new formulation in Section IV and explain the algorithm design in Section V. In Section VI, we generate a large number of cognitive radio system scenarios and show how often the utility can be improved. We conclude this paper with possible future work in Section VII.

II. Conventional Problem Formulation

In a wireless network, each user utilizes a channel (a segment of the radio spectrum) to transmit and to receive data. Primary users have higher priority in their licensed bands over secondary users. The latter can only employ those channels when they are not being used by the primary users and they must give up these channels whenever the primary users need them.

We define CRSAP according to [2]. For clarity, all the symbols used in the sequel are listed with their meanings in Table I. Without channel estimation, the interference range is defined as the outage interference range. Similar to the protocol model given in [16], the interference range is equal to the transmission range multiplied by $(1+\Delta)$, where $\Delta > 0$ is the guard zone specified by the protocol for data transmission. Hence a user can control its interference range precisely by adjusting its transmission power. Consider a set of homogeneous primary users $P_u = \{p_1, \ldots, p_G\}$ of size G, a set of homogeneous secondary users $S_u = \{s_1, \ldots, s_N\}$ of size N, and M orthogonal channels. We have the locations of the primary and secondary users in terms of x- and y-coordinates, namely, $X_p = [x(p_1), \ldots, x(p_G)]^T$ and $Y_p = [y(p_1), \ldots, y(p_G)]^T$ for the x- and y-coordinates of the primary users, respectively, and $X_s = [x(s_1), \ldots, x(s_N)]^T$ and $Y_s = [y(s_1), \ldots, y(s_N)]^T$ for the secondary users. Assume that every (primary or secondary) user has an omnidirectional antenna. It can control its transmission power and hence its interference range. Let $d(\phi, m)$ be the interference range of user ϕ with channel m, where $\phi \in P_u \cup S_u$. We have an interference range matrix for the primary users $R_p = [r^p_{g,m}]_{G\times M}$, where $r^p_{g,m} > 0$ indicates the interference range of Primary User g on Channel m and $r^p_{g,m} = 0$ means that Channel m is not used by Primary User g. We calculate the Euclidean distance between Users ϕ and γ ($\phi, \gamma \in P_u \cup S_u$) by $DIST(\phi,\gamma) = \sqrt{(x(\phi)-x(\gamma))^2 + (y(\phi)-y(\gamma))^2}$. After all primary users have decided their desired channels and the corresponding interference ranges (through controlling the transmission powers), the secondary users can then determine the *maximum* transmission powers (and also the interference ranges) so that they do not interfere with any primary users.[1] Due to hardware constraints, the interference range should be bounded, given by, for user ϕ and channel m

$$d_{\min} \le d(\phi, m) \le d_{\max}. \tag{1}$$

According to the locations and the interference ranges of both primary and secondary users, we can have the channel availability matrix $L = [l_{n,m} | l_{n,m} \in \{0,1\}]_{N\times M}$, where $l_{n,m} = 1$ means that Channel m is available for Secondary User n to use, i.e., the use of Channel m by s_n does not affect any primary users. Otherwise, $l_{n,m}$ is equal to zero. We also have the channel reward matrix $B = [b_{n,m}]_{N\times M}$, where $b_{n,m}$ characterizes the reward when Secondary User n adopts Channel m. As in [3] and [4], we characterize the reward as being proportional to the service coverage area by setting

$$b_{n,m} = f(d_s(n,m)) = d_s(n,m)^2 \tag{2}$$

throughout this paper, where $d_s(n,m)$ refers to the interference range of s_n for Channel m. Moreover, we describe the interference between the secondary users with the interference constraint matrix $C = [c_{n,k,m} | c_{n,k,m} \in \{0,1\}]_{N\times N\times M}$, where $c_{n,k,m} = 1$ implies that s_n will interfere with s_k if they both use Channel m. Otherwise, $c_{n,k,m}$ equals zero. By using Appendix I of [2], we compute L, B, and C, which are the necessary information to define the problem from the system environment data X_p, Y_p, X_s, Y_s, and R_p. Finally, the channel assignment matrix $A = [a_{n,m} | a_{n,m} \in \{0,1\}]_{N\times M}$ is used to indicate which channels are allowed to be utilized by the secondary users, where $a_{n,m} = 1$ means that Channel m is allocated to Secondary User n, and $a_{n,m} = 0$, otherwise. An assignment A is conflict-free if a secondary user is only assigned with channels which do not conflict with any other user. This can be described by

$$a_{n,m} + a_{k,m} \le 1, \quad \forall c_{n,k,m} = 1, 1 \le n,k \le N, 1 \le m \le M. \tag{3}$$

Moreover, due to hardware limitations, each cognitive radio interface should have a limit $C_{\max}$ on the maximum number

[1] It will be seen that the reward gained by User n on Channel m increases with the corresponding interference range. The larger the transmission range (and the interference range), the more subscribers it can serve. The reward seems to be maximized by maximizing the interference range.

TABLE I
DEFINITIONS OF SYMBOLS

Symbol	Definition
P_u (S_u)	Set of primary (secondary) users
p_i (s_i)	The ith primary (secondary) user
G (N)	Total number of primary (secondary) users
M	Total number of channels
X_p (Y_p)	x- (y-) coordinates of the primary users
X_s (Y_s)	x- (y-) coordinates of the secondary users
$d(\phi, m)$	Interference range of user ϕ with channel m
R_p	Interference range matrix for the primary users
$r^p_{g,m}$	Interference range of Primary User g on Channel m
$DIST(\phi, \gamma)$	Euclidean distance between Users ϕ and γ
$d_{\max}$ ($d_{\min}$)	Maximum (minimum) interference range
L	Channel availability matrix
$l_{n,m}$	Boolean variable indicating if Channel m is available for Secondary User n
B	Channel reward matrix
$b_{n,m}$	Reward of Secondary User n adopting Channel m
C	Interference constraint matrix
$c_{n,k,m}$	Boolean variable indicating if s_n interferes s_k when they use Channel m
A	Channel assignment matrix
$a_{n,m}$	Boolean variable indicating if Channel m is allocated to s_n
$C_{\max}$	Maximum number of channels equipped by each secondary user
$U(A)$	Total utility for A
Λ	Feasible solution set

of channels assigned [2], [17]. This can be expressed as

$$\sum_{m=1}^{M} a_{n,m} \leq C_{\max}, \quad \forall 1 \leq n \leq N. \tag{4}$$

We decide to maximize the reward gained from an assignment A represented by utility function $U(A)$. As in [2], [3], we express the utility as Max-Sum-Reward (MSR), Max-Min-Reward (MMR), or Max-Proportional-Fair (MPF), given by

$$U_{MSR}(A) = \sum_{n=1}^{N} \sum_{m=1}^{M} a_{n,m} \cdot b_{n,m}; \tag{5}$$

$$U_{MMR}(A) = \min_{1 \leq n \leq N} \sum_{m=1}^{M} a_{n,m} \cdot b_{n,m}; \tag{6}$$

$$U_{MPF}(A) = (\prod_{n=1}^{N} (\sum_{m=1}^{M} a_{n,m} \cdot b_{n,m} + 10^{-6}))^{\frac{1}{N}}, \tag{7}$$

respectively. MSR and MMR maximize the utilization of the whole network and that of the most disadvantaged user, respectively. MPF is for fairness. An assignment A is a solution to the problem. Those assignments which satisfy both Constraints (3) and (4) form the feasible solution set Λ. Mathematically, the conventional problem formulation is presented as $\max_{A \in \Lambda} U(A)$, where $U(A)$ can be $U_{MSR}(A)$, $U_{MMR}(A)$, or $U_{MPF}(A)$.

III. ANALYSIS ON ADJUSTABLE COVERAGE: TWO-SECONDARY USER SCENARIO

With X_p, Y_p, X_s, Y_s, and R_p, a central authority computes a feasible channel assignment for a cognitive radio system (i.e., a set of primary and secondary users in a given area) with maximum utility with a procedure as follows: (i) Collect system information, i.e., X_p, Y_p, X_s, Y_s, and R_p; (ii) Determine the maximum possible interference range (service coverage area) of each secondary user for every channel, i.e. $d_s(n, m), \forall 1 \leq n \leq N, 1 \leq m \leq M$, and the reward according to (2) (obtaining L and B); (iii) Determine the interference conflicting relationship among the secondary users (obtaining C); and (iv) Compute a channel assignment A with an optimization algorithm according to a certain objective. From this, we can see that the interference ranges $d_s(n, m)$ computed at Step (ii), which takes the maximum possible value in the range $[d_{\min}, d_{\max}]$, dictate the potential conflict of interference among the secondary users with C, which in turn limits the size of the solution space Λ.

Now we consider a scenario given in Fig. 1 which is common in a cognitive radio system. There are three primary users (p_1, p_2, and p_3) and two secondary users (s_1 and s_2). Assume that they all operate on the same channel (e.g., Channel m). Note that the primary and secondary users are regarded as base stations and they serve the mobile users located in their own service coverage areas. The larger the area, the more mobile users they serve. We consider an ideal situation that the interference ranges of two adjacent cells should not overlap. Otherwise, those mobile users located in the overlapping area would be interfered. The interference range of s_1 is limited by those of p_1 and p_2; and similarly, s_2 is influenced by p_1 and p_3. The maximum interference ranges of s_1 and s_2 are given by the dotted circles surrounding them. If the maximum interference ranges are adopted (for the purpose of maximizing the reward intuitively), s_1 and s_2 conflict with each other and they cannot be assigned to Channel m concurrently. Let d_{s1} and d_{s2} be the maximum interference ranges of s_1 and s_2, respectively. If we consider MSR, by (2), the total reward

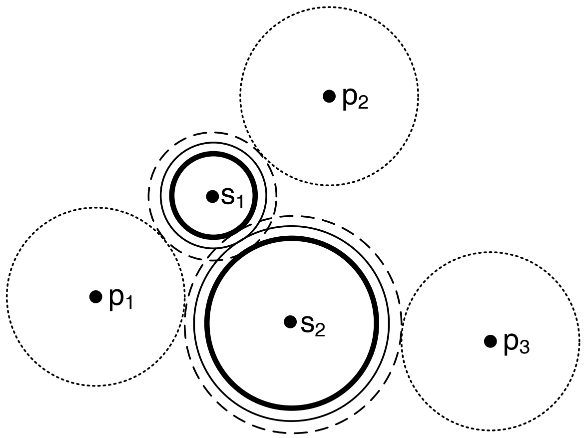


Fig. 1. A typical scenario with two secondary users in a sub-system of an infrastructure-based cognitive radio network.

gained by s_1 and s_2 together will be either $f(d_{s1})$ or $f(d_{s2})$, depending on whether Channel m is assigned to s_1 or to s_2. We try to show that it is possible to have an even higher total reward by not maximizing the interference ranges of s_1 and s_2. We consider several combinations of interference ranges for s_1 and s_2, i.e. (i) thick-solid-line circle for s_1 and dotted-line circle for s_2, (ii) dotted-line circle for s_1 and thick-solid-line circle for s_2, and (iii) thin-solid-line circles for both s_1 and s_2. Let d'_{s1} and d'_{s2} be their new interference ranges set according to any one of the combinations. We can see that s_1 and s_2 will not conflict with each other if any one of the combinations is chosen for setting their interference ranges. Hence they can be assigned with Channel m at the same time and the total reward may be higher as it is possible to have $f(d'_{s1})+f(d'_{s2}) > f(d_{s1})$ or $f(d'_{s1})+f(d'_{s2}) > f(d_{s2})$. Recall that the conventional formulation assumes the secondary users take up their maximum interference ranges, provided that their secondary users do not interfere with the primary users. However, if the interference ranges are not assigned their maximum values, it is possible to increase the system utility by the approach introduced above through spatial diversity, i.e., having the users (who are originally conflicting among themselves) operate on the same channel without conflict at the same time by carefully controlling their interference ranges. We will investigate further the possibility of increasing utility with spatial diversity for a two-secondary users scenario.

A. Analysis

Consider again the scenario given in Fig. 1. Let D be the distance between s_1 and s_2, and d'_{s1} and d'_{s2} be the adjusted interference ranges for s_1 and s_2, respectively. Fig. 2 shows all possible relative lengths of d'_{s1}, d'_{s2}, and D when d_{s1} and d_{s1} are given. Case I stands for the situation when $d_{s1}+d_{s2} \leq D$ while the rest are for $d_{s1}+d_{s2} > D$. By simple analysis, we obtain the feasible ranges of d'_{s1} and d'_{s2} as:

- Case I: $d'_{s1} = d_{s1}$ and $d'_{s2} = d_{s2}$;
- Case II: $\max(d_{\min}, D-d_{s2}) \leq d'_{s1} \leq \min(d_{s1}, D-d_{\min})$ and $\max(d_{\min}, D-d_{s1}) \leq d'_{s2} \leq \min(d_{s2}, D-d_{\min})$;

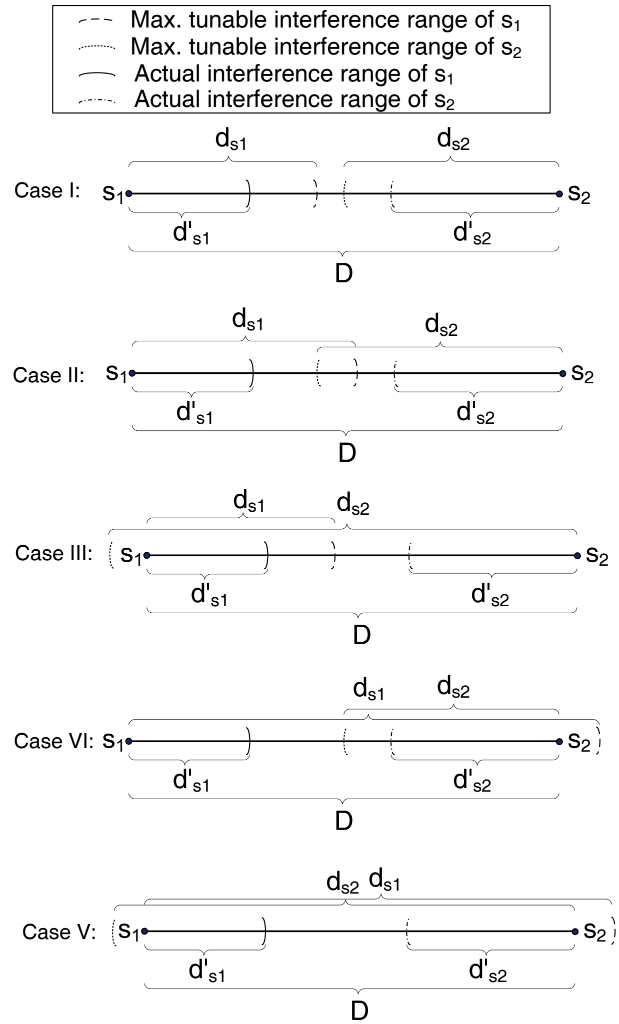


Fig. 2. Case study of variable interference ranges of secondary users.

- Case III: $d_{\min} \leq d'_{s1} \leq \min(d_{s1}, D-d_{\min})$ and $\max(d_{\min}, D-d_{s1}) \leq d'_{s2} \leq D-d_{\min}$;
- Case IV: $\max(d_{\min}, D-d_{s2}) \leq d'_{s1} \leq D-d_{\min}$ and $d_{\min} \leq d'_{s2} \leq \min(d_{s2}, D-d_{\min})$; and
- Case V: $d_{\min} \leq d'_{s1}, d'_{s2} \leq D-d_{\min}$.

To summarize, If both s_1 and s_2 operate at the same time, the new interference ranges of s_1 and s_2 should be adjusted in the following ranges:

$$\left.\begin{array}{l}\max(d_{\min}, D-d_{s2}) \leq d'_{s1} \leq \min(d_{s1}, D-d_{\min}) \\ \max(d_{\min}, D-d_{s1}) \leq d'_{s2} \leq \min(d_{s2}, D-d_{\min})\end{array}\right\} \quad (8)$$

We can find the optimal d'_{s1} and d'_{s2} by maximizing $U(d'_{s1}, d'_{s2}) = (d'_{s1})^2 + (d'_{s2})^2$ subject to (8). For maximum utility, we have $d'_{s1} + d'_{s2} = D$. Thus, (8) becomes $\max(d_{\min}, D-d_{s2}) \leq d'_{s1} \leq \min(d_{s1}, D-d_{\min})$. By simple analysis, we obtain the optimal interference ranges given in Table II. Note that it is not trivial to extend results for the two-secondary user scenario (i.e. (8) and Table II) to those with multiple secondary users. For the former, two secondary users form a line and all ranges interact in the one-dimensional space. This makes the analysis tractable. For the latter, the analysis needs to be done in the two-dimensional space and it

TABLE II
OPTIMAL INTERFERENCE RANGES FOR THE TWO-SECONDARY USER SCENARIO.

Case	Conditions	Optimal interference range
A	$(d_{\min} > D - d_{s2})$ and $(d_{s1} > D - d_{\min})$	$\arg\max_{d'_{s1}} F(d'_{s1}) = d_{\min} = D - d_{\min}$
B	$(d_{\min} > D - d_{s2})$ and $(d_{s1} \leq D - d_{\min})$	$\arg\max_{d'_{s1}} F(d'_{s1}) = \begin{cases} d_{\min} & \text{if } d_{s1} < D - d_{\min} \\ d_{\min} \text{ or } d_{s1} & \text{if } d_{s1} = D - d_{\min} \end{cases}$
C	$(d_{\min} \leq D - d_{s2})$ and $(d_{s1} > D - d_{\min})$	$\arg\max_{d'_{s1}} F(d'_{s1}) = \begin{cases} D - d_{\min} & \text{if } d_{s2} < D - d_{\min} \\ D - d_{s2} \text{ or } D - d_{\min} & \text{if } d_{s1} = D - d_{\min} \end{cases}$
D	$(d_{\min} \leq D - d_{s2})$ and $(d_{s1} \leq D - d_{\min})$	$\arg\max_{d'_{s1}} F(d'_{s1}) = \begin{cases} D - d_{s2} & \text{if } D > d_{s1} + d_{s2} \text{ and } d_{s1} \geq d_{s2} \\ D - d_{s2} & \text{if } D < d_{s1} + d_{s2} \text{ and } d_{s1} < d_{s2} \\ d_{s1} & \text{if } D < d_{s1} + d_{s2} \text{ and } d_{s1} \geq d_{s2} \\ d_{s1} & \text{if } D > d_{s1} + d_{s2} \text{ and } d_{s1} < d_{s2} \\ d_{s1} \text{ or } D - d_{s2} & \text{if } D = d_{s1} + d_{s2} \end{cases}$
Remark: $F(d'_{s1}) \triangleq (d'_{s1})^2 + (D - d'_{s1})^2$		

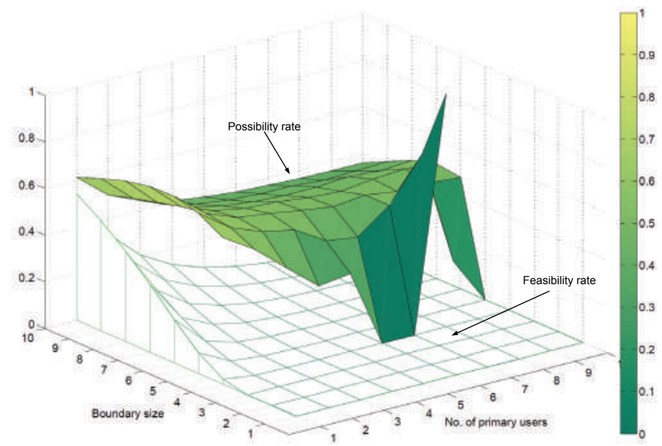


Fig. 3. Feasibility and possibility rates.

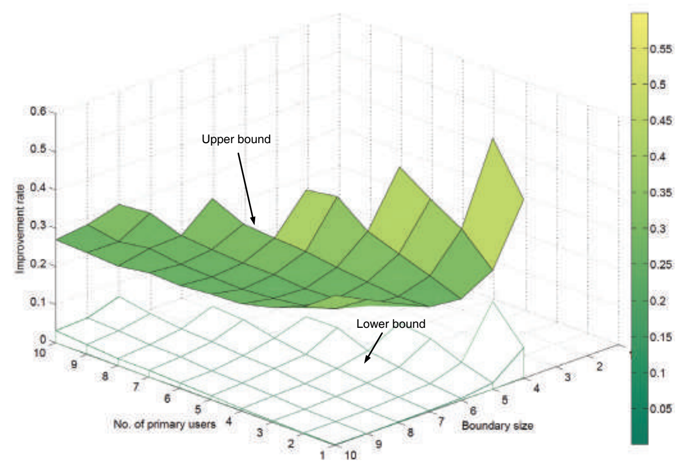


Fig. 4. Lower and upper bounds of improvement rates.

becomes much more complex. In the next section, we will show that the new formulation to the problem with range adjustability is computationally intractable.

B. Possibility of Utility Improvement

As secondary users cannot affect primary users at all, the shorter the distance between a primary and a secondary user, the smaller the flexibility (i.e., the adjustable interference range) the secondary user can have. Thus, the density of primary users around the pair of secondary users may influence the possibility of utility enhancement. By simulation, we study a scenario where two secondary users and G primary users are deployed in a given l_b-by-l_b square area. The impact of the primary user density can be realized by controlling G and l_b. We create one million such scenarios for different combinations of values of G and l_b, each of which takes integer values ranging from 1 to 10, respectively.

We try to adjust d'_{s1} and d'_{s2}, within (8) in order to further improve the utility. The feasible scenarios are obtained by excluding the infeasible cases, namely, cases where not all secondary users can function, on $d_{s1} < d_{\min}$, $d_{s2} < d_{\min}$, or $D < 2d_{\min}$. We define the *feasibility rate* as the ratio of the number of feasible scenarios to that of all generated scenarios. The lower curtain mesh plot in Fig. 3 shows the feasibility rate for each combination of G and l_b. We can see that the lower the density of primary users , the higher the feasibility rate. Among the feasible scenarios, some belong to Cases II–V of Fig. 2 which may be enhanced by adjusting d'_{s1} and d'_{s2}. We define the *possibility rate* as the ratio of the total number of cases belonging to Cases II–V to all feasible scenarios (Cases I–V). The upper surface plot in Fig. 3 shows the possibility rates for different combinations of G and l_b.[2] The possibility rate is higher than 0.4 and this shows that a certain (not small) portion of actual scenarios can be improved for higher utility with the new approach.

Among the feasible scenarios, we are interested in those corresponding to Cases II–V of Fig. 2 because the utility may be improved by adjusting d'_{s1} and d'_{s2}. A successfully improved scenario refers to one with the condition $U(d'_{s1}, d'_{s2}) > U(\max(d_{s1}, d_{s2}))$. We define the *improvement rate* as the ratio of the number of successfully improved scenarios to the total number of scenarios. Fig. 4 gives upper and lower bounds of the improvement rate corresponding to the set of

[2]Low feasibility rate implies that the denominator of the corresponding possibility rate is small. The possibility rate with low feasible rate is not conclusive as the number of cases belonging to Cases I-V is too small. Thus the upper surface plot fluctuates when the number of primary users is larger than three and the boundary size l_b is smaller than six.

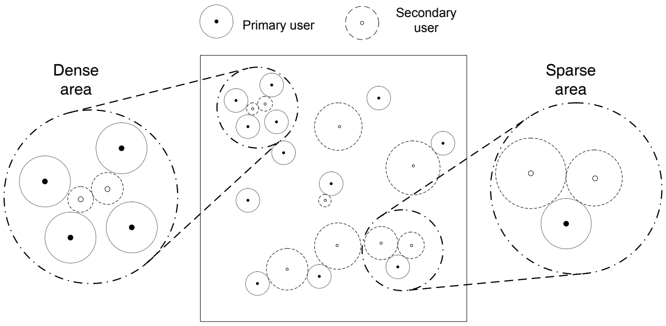


Fig. 5. Dense and sparse area scenarios.

scenarios used in Fig. 3.[3] The upper bound is the optimal value computed by those equations listed in Table II while the lower bound is derived by taking the lower limit of the secondary users' interference ranges given in Section III-A. Therefore, the possibility of utility enhancement by controlling the interference range may be large.

In a real cognitive radio system, we may have many more secondary users, e.g., in Fig. 5. When three or more secondary users interact with one another, the calculations for the optimal secondary user interference ranges for overall maximum utility become much more complex than those given in the previous subsections. However, a simulated scenario above can be considered as a section of an actual cognitive radio system with only two secondary users. A scenario with high (low) primary user density may correspond to the dense (sparse) area in Fig. 5. Therefore, our analysis and results can be considered as an approximation of a section in an actual system. To account for the general situation, our algorithm proposed in Section V can handle scenarios with multiple secondary users and we will verify its performance in Section VI.

IV. New Problem Formulation with Power Control

We propose a new formulation of CRSAP using the same system input parameters (i.e. X_p, Y_p, X_s, Y_s, and R_p) but we allow adjustable interference range. We define $I_{>0}(\xi) = 1$ if $\xi > 0$. Otherwise, $I_{>0}(\xi) = 0$. We also define $R_s = [r^s_{n,m}]_{N\times M}$, where $r^s_{n,m} > 0$ denotes the interference range of s_n on channel m and $r^s_{n,m} = 0$ means that channel m is not used by s_n. The problem can now be formulated as maximizing $U(R_s)$ subject to

$$r^s_{n,m} \begin{cases} \leq DIST(s_n, p_g) - r^p_{g,m} & \text{if } r^p_{g,m} > 0 \\ < \infty & \text{otherwise,} \end{cases},$$
$$\forall 1 \leq n \leq N, 1 \leq g \leq G, 1 \leq m \leq M, \quad (9)$$

$$r^s_{n,m} + r^s_{k,m} \leq DIST(s_n, s_k), \forall 1 \leq n, k \leq N, 1 \leq m \leq M, \quad (10)$$

$$r^s_{n,m} \in [d_{\min}, d_{\max}] \cup \{0\}, \forall 1 \leq n \leq N, 1 \leq m \leq M, \quad (11)$$

$$\sum_{m=1}^{M} I_{>0}(r^s_{n,m}) \leq C_{\max}, \forall 1 \leq n \leq N. \quad (12)$$

[3] As results associated with low feasibility rates are not conclusive, we remove the portions of the plots in Fig. 3 with the corresponding feasibility rate smaller than 0.001 for clarity.

Eq. (9) prevents the secondary users from interfering with any primary users and this is originally implied by the construction of matrix C implicitly. Eq. (10) prevents the interference ranges of any pair of secondary users from overlapping and this was conveyed from (3). Eq. (11) defines the physical operating ranges and (12) is similar to (4). Eqs. (5), (6), and (7) becomes

$$U_{MSR}(R_s) = \sum_{n=1}^{N}\sum_{m=1}^{M} b_{n,m}, \quad (13)$$

$$U_{MMR}(R_s) = \min_{1\leq n\leq N} \sum_{m=1}^{M} b_{n,m}, \quad (14)$$

$$U_{MPF}(R_s) = (\prod_{n=1}^{N}(\sum_{m=1}^{M} b_{n,m} + 10^{-6}))^{\frac{1}{N}}, \quad (15)$$

respectively, where $b_{n,m} = (r^s_{n,m})^2$ according to (2). Note that, for those channels (e.g., m) which are not assigned to the users (e.g., s_n), the corresponding interference ranges (e.g., $r^s_{n,m}$) are set to zero. Thus we do not need to consider the assignment $a_{n,m}$ in (13)–(15) as in (5)–(7). We call this new formulation power-controlled cognitive radio spectrum allocation problem (PC-CRSAP).

CRSAP has a discrete solution space with boolean variables, where "0" indicates that a channel is not assigned to a particular radio interface while "1" means that a channel is assigned to one with a pre-determined interference range. Note that the pre-determined interference ranges are given problem data computed by a greedy approach [2], which chooses a value in $[d_{\min}, d_{\max}]$ as large as possible without affecting any primary users for each range before considering the conditions of other secondary users. These ranges are not variables of CRSAP. Although PC-CRSAP is formulated based on the same system information, it considers the interference ranges as variables. In other words, the ranges are not pre-determined and computed by solving the optimization problem. When an interference range takes the "0" value for a particular channel, it still means the corresponding interface is not in use. We extend the discrete point "1" in CRSAP to a range $[d_{\min}, d_{\max}]$ in PC-CRSAP. The new formulation has a continuous solution space with some separate points. In general, the physical meanings of the objectives and constraints of the two formulations are the same, but the new one provides more flexibility of utilizing resources; CRSAP is a special case of PC-CRSAP by setting $d_{\min} = d_{\max}$, whose value equals the pre-determined value determined by the greedy approach in [2].

The problem is non-convex, and the analysis and derivation of upper bounds are non-trivial. We demonstrate the difficulty of the problem via duality in the appendix, which also provides a framework for further analysis. In order to formally characterize the computational complexity of PC-CRSAP, we have the following proposition.

Proposition 1. *PC-CRSAP is NP-hard.*

Proof: We prove the result by contradiction. Assume that PC-CRSAP can be solved in polynomial time. Consider that we try to solve the conventional cognitive radio spectrum allocation problem (CRSAP). The difference between the two problems, PC-CRSAP and CRSAP, is that the former allows

```
procedure repair(A)
  A' := A
// For constraint (3)
  for n := 1 to N do
    for k := 1 to N do
      for m := 1 to M do
        if (c_{n,k,m} == 1) AND (a_{n,m} == 1) AND (a_{k,m} == 1)
          Randomly generate a number i ∈ [0,1]
          if i < 0.5
            a'_{n,m} := 0
          else
            a'_{k,m} := 0
          end if
        end if
      end for
    end for
  end for
// For constraint (4)
  for n := 1 to N do
    z := a_{n,1} + ... + a_{n,M}
    if z > C_max
      Randomly generate (z − C_max) numbers i_1, ..., i_(z−C_max) such that a_{n,i_1} = ... = a_{n,i_(z−C_max)} = 1
      a'_{n,i_1} := 0
        ⋮
      a'_{n,i_(z−C_max)} := 0
    end if
  end for
  return A'
end procedure
```

Fig. 6. Pseudocode of the $repair$ operator.

each $r^s_{n,m}$ to take any values in the range $[d_{\min}, d_{\max}] \cup \{0\}$ while the latter restricts $r^s_{n,m}$ to $\{d_{\max}\} \cup \{0\}$ (we can assign "1" to $d_{\max}$ and thus CRSAP becomes a combinatorial problem). Hence we can restate CRSAP as PC-CRSAP by setting $d_{\min} = d_{\max}$ in the formulation. However, CRSAP is *NP*-hard, as proved in [2], and PC-CRSAP cannot be easier than CRSAP. This contradicts our assumption. This proves the result. ■

V. Algorithm Design

We employ a two-phase structure to design the algorithm. In Phase I, we aim to solve CRSAP. Then we modify the solution computed in Phase I to solve PC-CRSAP in Phase II. Due to space limitations, we will briefly go through Phase I but focus on Phase II. More details about Phase I can be found in [4].

A. Phase I

A solution of CRSAP is a channel assignment matrix A, whose entities are 0/1 indicators specifying whether particular channels are assigned to certain secondary users. Such a matrix A gives the complete picture for all channels and (secondary) users. We decide to find an A which can maximize the objective function $U(A)$, by iteratively giving different 0/1 combinations for A. However, this matrix representation contains many redundancies. We only consider those entities in A whose channels are available to the users. This makes the search more efficient as the infeasible solutions will not be considered by the algorithm anymore [3], [4]. Whenever a new solution A is produced by the algorithm, we check and repair any constraint violations by invoking the "$repair$" procedure to generate a feasible A' from A, i.e. $A' = repair(A)$. $repair$ consists of two parts, each of which is used to tackle one of the two constraints. The pseudocode of the $repair$ operator is shown in Fig 6.

There are four types of elementary reactions defined in CRO, namely, on-wall ineffective collision, decomposition, inter-molecular ineffective collision, and synthesis. Each of them is used to manipulate solutions held by the agents, i.e., molecules. We explore the solution space and locate the global optimum through a random sequence of elementary reactions according to certain rules.

Here we give the outline of the algorithm. We basically follow the design framework described in [9] to develop a CRO-based algorithm to solve CRSAP. A flow chart of the algorithm can be found in [4]. In the initialization, we create the initial set of molecules with size equal to *PopSize* and their molecular structures are solutions in the matrix representation by randomly setting every bit. We pass each solution to $repair$ to ensure it is feasible. Then the objective function is evaluated and the corresponding values are the *PE* of the molecules. The initial *KE* of every molecule is set to the value of *InitialKE*. In each iteration, we first convert the solutions from the matrix to vector form with the *M2V* operator. Then we decide whether a uni-molecular or an inter-molecular reaction is carried out in the iteration by comparing a random number $h \in [0, 1]$ with *MoleColl*. We select an appropriate subset of molecules to undergo an elementary reaction determined by the decomposition criterion or the synthesis criterion (depending on whether the elementary reaction is uni-molecular or inter-molecular). Next we convert the solutions back to the matrix form with the *V2M* operator. After repairing, the objective functions of the solutions are evaluated. The iteration process continues until a stopping criterion is satisfied. We output the best-so-far solution in the final stage. More information about the elementary reactions can be found in [14] and their detailed implementations for CRSAP are also given in [4].

B. Phase II

In this phase, we try to modify the solution computed in Phase I, i.e., $A = [a_{n,m}]_{N\times M}$, to $R_s = [r^s_{n,m}]_{N\times M}$ so as to increase the utility by adjusting interference ranges of the secondary users. Since interference ranges are channel-specific (i.e. they are independent, and thus, they can be tuned separately), we consider a"[secondary user, channel]" combination as an adjustment unit. The sooner a unit is adjusted, the higher flexibility it has, i.e., the larger the interference range that a secondary user can choose for the channel. An adjustment unit considered later will have more restrictions on its interference range as more units nearby have been fixed. Therefore, the order in which adjustment units are considered affects the overall utility but it is hard to determine the optimal order. Thus, we randomly generate an $N \times M$ sequence of adjustment units and follow this sequence to tune the interference ranges. For each adjustment unit, we first check whether the channel has been assigned to the secondary user. If so, we proceed to the next unit. Otherwise, we try to turn on the channel for the user with maximum interference range which will not affect any other users. If this maximum range does not meet the hardware constraint, i.e. (1), we proceed to the next unit. We repeat the above process until all units have been adjusted. Note that not all $N \times M$ adjustment units need be considered as those already activated units will be skipped. However, generating a complete $N \times M$ random sequence makes the implementation simpler.

The above adjustment process is simple and fast. We do not need to make any changes to those adjusted user-channel combinations. Although it is possible to backtrack and obtain even higher utility by reducing the interference ranges of some adjusted units so as to increase the ranges of other units, e.g. using the thin-solid-line circles for both s_1 and s_2 in Fig. 1, making changes to adjusted units will affect other adjusted units too. The situation will become far more complex. We leave this further enhancement for future work.

It is worth mentioning that this two-phase algorithmic structure not only works for CRO but also other algorithms. In Phase I, we can apply any existing algorithm to solve CRSAP and we propose to use CRO in this paper because CRO can give superior performance (this will be demonstrated in the next section). In Phase II, we can modify any solution obtained (using CRO or other algorithms) in Phase I to generate an improved solution for PC-CRSAP. We can consider this algorithmic structure as a framework to tackle PC-CRSAP.

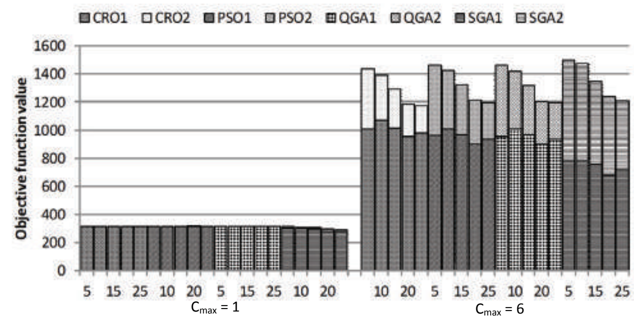
VI. Simulation Results

In this section, we will compare the performance of CRO with three other evolutionary algorithms, i.e. CGA, QGA, and PSO, on solving both CRSAP and PC-CRSAP. They are chosen because they are adopted to solve a similar problem (which is CRSAP addressed in this paper but without Constraint (4)) in [3]. They are all implemented with the same solution-space-reduction (i.e. conversion between matrix and vector representations of solutions) and constraint-violation-removal (i.e. the $repair$ operator) techniques as CRO as described in Section V. We do not consider the heuristic graph coloring approach proposed in [2] in our comparison since [3] has already shown that the evolutionary algorithms have much better performance than the graph coloring method. Thus, we expect CRO to also outperform the heuristic approach if CRO outperforms CGA, QGA, and PSO.

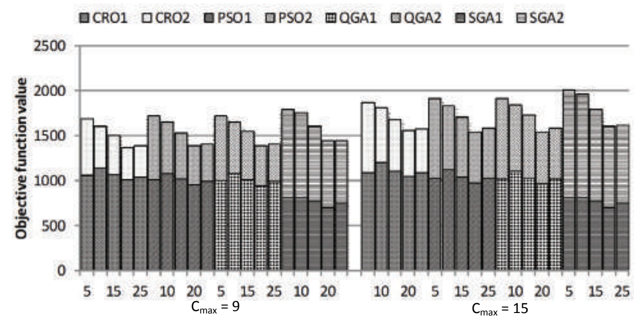
To have fair comparisons of performance over various optimization strategies, we create 50 random topologies of primary and secondary users as the set of benchmark problems. Assume that there are 20 secondary users and 20 orthogonal channels, similar to an instance given in [3]. We deploy G primary users in a region of area 15×15 square units, where $G \in \{5, 10, 15, 20, 25\}$, and each primary user has a constant interference range, equal to 2, for all channels, i.e. $d_p(n,m) = 2, \forall 1 \le n \le 20, 1 \le m \le 20$. The minimum and maximum interference ranges of the secondary users are 1 and 4, respectively, i.e. $d_{\min} = 1$ and $d_{\max} = 4$. We create 10 problem instances for each value of G (and hence there are 5×10 problem instances in total).

A. CRSAP

We first compare the performance of the algorithms for CRSAP, i.e., the phase I of the algorithmic structure. We follow [2] to determine the interference range of the secondary users $d_s(n,m)$ and set $b_{n,m} = d_s(n,m)^2$. Each problem



(a) $C_{\max}$ equal to 1 and 6



(b) $C_{\max}$ equal to 9 and 15

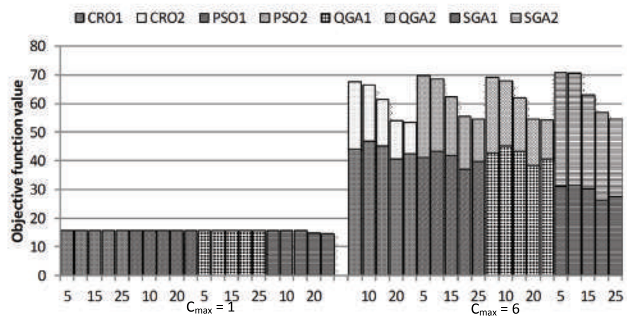
Fig. 7. Performance of the algorithms for both phases I and II on solving MSR.

instance is associated with a set of channel availability matrix L, channel reward matrix B, and interference constraint matrix C. We compare the performance of the algorithms with the 50 benchmarks. The parameters of CRO are set as follows: *PopSize* $= 20$, *KELossRate* $= 0.2$, *InitialKE* $= 800$, *MoleColl* $= 0.5$, $\alpha = 3000$, and $\beta = 10$. For CGA, QGA, and PSO, the parameters are configured according to [2]. As there are no theoretical results on how to tune the parameter values of evolutionary algorithms and the parameter adjustments are highly dependent on the experience and preferences of users [18], the parameter values of CRO are chosen after we have performed several trial runs of the simulation. For CGA, QGA, and PSO, the parameters are set according to [3].

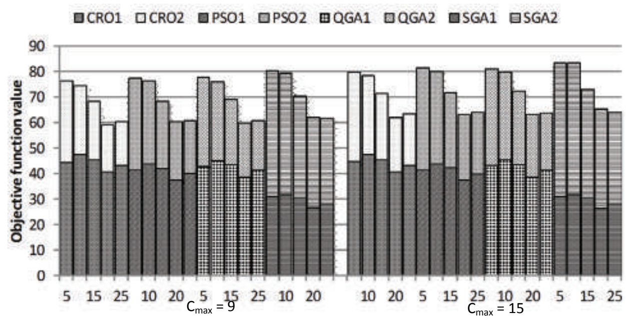
We follow the pseudo-codes given in [3] to develop CGA, QGA, and PSO and all simulation codes (including CRO) are programmed in Java. All simulations are run on the same computer with Intel Core i5-23102.9Ghz and 8GB of RAM operating on Windows 7, 64 bit and JRE 7u5. As in [3], the stopping criterion is when the maximum number of function evaluations, equal to 6000, is reached (i.e. 300 generations for CGA, QGA, and PSO).

We investigate the impact on the number of allowed assigned channels to the users by changing the values of $C_{\max}$ from 1 to 20. The smaller the value of $C_{\max}$, the smaller the number of channels allocated to the users, and thus, the smaller the objective function values. The results for the objective MSR are shown in Fig. 7[4], where the results computed in the first phase are indicated by the acronyms of the algorithms followed by "1". Each block of bars corresponds to a particular

[4]Due to space limitations, selected cases with $C_{\max}$ equal to 1, 6, 9, and 15 are given.



(a) $C_{\max}$ equal to 1 and 6



(b) $C_{\max}$ equal to 9 and 15

Fig. 8. Performance of the algorithms for both phases I and II on solving MPF.

value of $C_{\max}$. In each block, the numbers 5, 10, 15, 20, and 25 indicate the number of primary users. Each bar represents the average of 100 different simulation runs (10 times for each of the 10 topologies). For each algorithm, in general, the utility decreases with the number of primary users. It can be seen that the secondary users tend to have smaller allowed inference ranges when there are more primary users nearby. Moreover, when the secondary users can be equipped with more channels (indicated by $C_{\max}$), the utility will increase, saturating after a certain value of $C_{\max}$ (i.e., $C_{\max} = 15$ in the simulation given in Fig. 7). This means that the algorithms cannot improve the performance further when more channels are allocated to the users. In other words, the algorithms can only result in sub-optima with similar utility even when higher flexibility is allowed. When we compare the performance among the algorithms, CRO outperforms the others in all cases. Similar trends are also obtained for MPF and the results are given in Fig. 8. We have also performed other simulations and comparisons (e.g., for MMR) and they are reported in [4].

For computational time, each run of CRO takes around 0.1s on the average, which is sufficiently short when compared with the period of system parameter changes (i.e., changes in L, A, and B) in a static network environment. For the other algorithms, PSO, QGA, and SGA take 0.8s, 0.4s, and 0.2s, respectively. We can see that CRO takes much shorter time to compute the results.

B. PC-CRSAP

Next we examine how the interference range adjustment improves the overall utility in general situations with multiple primary and secondary users. Recall that the significance of Phase II determined by the density of the secondary users with respect to the primary users. Here we assume that the total number of secondary users is fixed ($N = 20$). We study the improvement of Phase II over Phase I by varying the number of primary users ($G = 5, 10, 15, 20, 25$) in a fixed area with size 15×15. The rest of the settings are the same as those stated in Section VI-A.

The simulation results for MSR are shown in Fig. 7, where the improvements due to Phase II are indicated with the acronyms of the respective algorithms followed by “2” and they are stacked with the results from Phase I for the corresponding cases. With the two-phase structure, Phase II always results in equal or better performance than Phase I. For any algorithms, when $C_{\max}$ increases, the secondary users have more channels to choose from, and thus, they have higher probability to adjust their interference ranges with better utility by spatial diversity. Our proposed heuristic in Phase II is independent of the algorithm used in Phase I. In general, Phase II improves the utilities dramatically when compared with those obtained in Phase I. All algorithms have similar performance after Phase II, even though they may not perform very well in Phase I, e.g., SGA1. We can see that Phase II is very effective, even for the inferior solutions obtained in Phase I. Similar results for MPF can be found in Fig. 8. Moreover, the overhead of Phase II is negligible as the average additional computation time due to Phase II is less than 1% of the overall computation. In conclusion, Phase II can improve the performance and the improvement decreases with the density of the secondary users relative to the primary users.

VII. CONCLUSION

One important problem in cognitive radio is allocating unused frequency channels to the unlicensed users effectively. In this paper, we develop an effective algorithm to tackle this problem. First, we propose a CRO-based algorithm for CRSAP. CRO is a chemical reaction-inspired metaheuristic for general optimization. By designing several operators under the CRO framework, we make CRO capable of generating good solutions which satisfy the constraints of CRSAP. We also consider the hardware constraint of limiting the maximum number of channels for a user. Simulation results show that CRO outperforms other proposed algorithms for CRSAP dramatically. Moreover, our analysis in Section III with the consideration of two secondary users, shows that we can further improve the utility by appropriately adjusting the secondary users' transmission power. The conventional formulation (i.e., CRSAP) assumes that a secondary user can be either on (with its maximum interference range) or off on a particular channel. We provide a new formulation (i.e., PC-CRSAP) to the problem with the consideration of adjustable interference ranges. In fact, CRSAP and PC-CRSAP take the same set of problem information and they are identical except that a solution in the new formulation is allowed to take real values with given ranges while it is restricted to discrete 0-1 values in the conventional formulation. Hence CRSAP is a special case of PC-CRSAP. To address PC-CRSAP, we propose a two-phase algorithmic framework, which can improve the solutions for CRSAP computed by *any* algorithm for PC-CRSAP. We propose to solve CRSAP with CRO in Phase I and modify this result by adjusting the interference ranges in Phase II, which is inspired by the analysis with two secondary users. Our two-phase algorithm performs very well even when the problem contains more secondary users. Simulation results show that results obtained from PC-CRSAP can be further enhanced through spatial diversity even for general cases with more than two secondary users significantly. This also says that a *wiser* formulation of the problem can improve system utility. Moreover, the heuristic proposed in Phase II can effectively improve those solutions which are originally inferior in Phase I. In general, our new formulation to the problem can facilitate better understanding of the system and stimulate further research.

In the future, we will develop tailor-made CRO operators for our problem to get even better performance. We will also work out a more intelligent adjustment procedure in Phase II for even better performance.

Appendix

We focus on the convexity of the new formulation PC-CRSAP. We inspect the constraints, and then the objective functions.

The constraints of the problem are (9), (10), (11), and (12). The variables are $r^s_{n,m}$, for $1 \leq n \leq N$ and $1 \leq m \leq M$. As $r^p_{g,m}$, $DIST(s_n, p_g)$, and $DIST(s_n, s_k)$ are constants, Constraints (9) and (10) are linear inequalities. Constraint (11) defines the feasible range of values for each $r^s_{n,m}$. With $0 \leq d_{\min} \leq d_{\max}$, unless $d_{\min}$ is equal to zero, $r^s_{n,m}$ is defined on a non-convex set. However, it is hardly possible to produce a base station with $d_{\min}$ arbitrarily close to zero. Moreover, together with Constraint (11), Constraint (12) states that, for each n, the number of $r^s_{n,m} \in [d_{\min}, d_{\max}]$ is less than or equal to $C_{\max}$ for all m. Assume that we relax Constraint (11) to be

$$r^s_{n,m} \in [0, d_{\max}], \quad \forall 1 \leq n \leq N, 1 \leq m \leq M. \tag{16}$$

Then Constraint (12) is converted to

$$\sum_{m=1}^{M} I_{\geq d_{\min}}(r^s_{n,m}) \leq C_{\max}, \quad \forall 1 \leq n \leq N. \tag{17}$$

We consider each term in the left-hand side of Inequality (17) and give its plot in Fig. 9. Let $f(r^s_{n,m}) = I_{\geq d_{\min}}(r^s_{n,m})$. We look at its epigraph $\text{epi} f = \{(r^s_{n,m}, \mu) : r^s_{n,m} \in [0, d_{\max}], \mu \in \mathbb{R}, \mu \geq f(r^s_{n,m})\}$, which is shaded in Fig. 9. We can see that f is non-convex as its epigraph is not a convex set. Thus, Constraint (17) is non-convex, so as the original Constraint (12).

Therefore, the domain of the problem defined by (9)–(12) is not a convex set.

Next we inspect the objective functions, given by (13), (14), and (15).

Since we maximize U_σ, where $\sigma \in \{MSR, MMR, MPF\}$, we can simply consider

$$\text{minimize} \quad -U_\sigma((r^s_{n,m})_{N\times M}). \tag{18}$$

Since each second partial derivative of $-U_\sigma$ is negative, clearly $-U_\sigma$ is non-convex.

The whole problem is non-convex and we cannot solve the problem effectively with the conventional convex optimization tools.

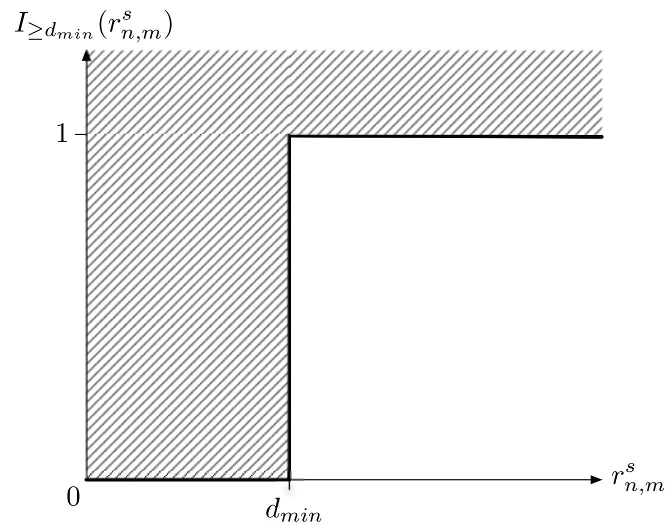


Fig. 9. Function $I_{\geq d_{\min}}(r^s_{n,m})$ and its epigraph.

Now we try to determine an upper bound of the problem by taking the dual. To simplify the representation, we define

$$A = (a_{n,m})_{N\times M},$$

where

$$a_{n,m} \triangleq \begin{cases} \min_g (DIST(s_n, p_g) - r^p_{g,m}) & \text{if there exists } g \text{ with } r^p_{g,m} > 0, \\ \infty & \text{otherwise,} \end{cases} \tag{19}$$

and

$$B = (b_{i,n})_{\binom{N}{2}\times N},$$

where $b_{i,n} \in \{0, 1\}$. Each row of B contains two different elements equal to 1 and the rest are 0. We also define

$$C = (c_{i,m})_{\binom{N}{2}\times M},$$

where, for all m, $c_{i,m} \triangleq DIST(s_n, s_k)$ if the ith row of B has $b_{i,n} = b_{i,k} = 1$, $\mathcal{D} \subset \mathbb{R}_{N\times M}$ such that each element in $\mathcal{D}$ has a value in the set $[d_{\min}, d_{\max}] \cup \{0\}$, and $E = [e_1, e_2, \ldots, e_N]^T$, where $e_n = C_{\max}, 1 \leq n \leq N$. With abuse of notation, we define a function $I : \mathbb{R}_{N\times M} \to \mathbb{R}_{N\times 1}$ such that $I(R_s)$ returns a vector of $\sum_{m=1}^{M} I_{\geq d_{\min}}(r^s_{n,m})$ for all n. For MSR (i.e., (13)), we have

$$\underset{R_s}{\text{minimize}} \quad -\text{Tr}(R_s^T R_s) \tag{20a}$$

$$\text{subject to} \quad R_s \preceq A, \tag{20b}$$

$$BR_s \preceq C, \tag{20c}$$

$$R_s \in \mathcal{D}, \tag{20d}$$

$$I(R_s) \preceq E, \tag{20e}$$

where Tr is the trace operator and $\preceq$ is element-wise $\leq$. Let $\Gamma = (\gamma_{n,m})_{N\times M}$, $\Lambda = (\lambda_{i,n})_{\binom{N}{2}\times M}$, and $\Psi = (\psi_n)_{N\times 1}$ be the Lagrangian multipliers for (20b), (20c), and (20e), respectively. The Lagrangian $L : \mathbb{R}_{N\times M} \times \mathbb{R}_{N\times M} \times \mathbb{R}_{\binom{N}{2}\times N} \times$

$\mathbb{R}_{N\times 1} \to \mathbb{R}$ associated with Problem (20) is

$$L(R_s,\Gamma,\Lambda,\Psi) = -\operatorname{Tr}(R_s^T R_s) + \operatorname{Tr}(\Gamma^T(R_s - A)) + \operatorname{Tr}(\Lambda^T(BR_s - C)) + \Psi^T(I(R_s) - E). \quad (21)$$

The dual function $g : \mathbb{R}_{N\times M} \times \mathbb{R}_{\binom{N}{2}\times N} \times \mathbb{R}_{N\times 1} \to \mathbb{R}$ is

$$\begin{aligned} g(\Gamma,\Lambda,\Psi) &= \inf_{R_s\in\mathcal{D}} L(R_s,\Gamma,\Lambda,\Psi) \\ &= -\operatorname{Tr}(\Gamma^T A) - \operatorname{Tr}(\Lambda^T C) - \Psi^T E + \\ &\inf_{R_s\in\mathcal{D}} \left(-\operatorname{Tr}(R_s^T R_s) + \operatorname{Tr}((\Gamma + B^T\Lambda)^T R_s) + \Psi^T I(R_s)\right). \end{aligned} \quad (22)$$

Once the infimum part of (22) has been solved, we can easily determine a lower bound of (20) (i.e., a upper bound of the original formulation) by solving the dual problem

$$\begin{aligned} \underset{\Gamma,\Lambda,\Psi}{\text{maximize}} \quad & g(\Gamma,\Lambda,\Psi) && (23a) \\ \text{subject to} \quad & \Gamma \succeq 0, && (23b) \\ & \Lambda \succeq 0, && (23c) \\ & \Psi \succeq 0, && (23d) \end{aligned}$$

which is a linear program. However, we cannot solve the infimum part of (22) easily due to the non-convex nature of $\mathcal{D}$ and $I(R_s)$. We cannot obtain an upper bound by duality for MSR, and the situations for MMR and MPF are similar. However, we can obtain trivial bounds for MSR, MMR, and MPF as $\sum_{n=1}^{N}\sum_{m=1}^{M} d_{\max}^2$, $\sum_{m=1}^{M} d_{\max}^2$, $(\prod_{n=1}^{N}(\sum_{m=1}^{M} d_{\max}^2 + 10^{-6}))^{\frac{1}{N}}$, respectively, but they may not be very useful to evaluate the results obtained by the heuristics proposed in the paper.

To conclude, the problem is very complex and there is no easy way to perform analysis or to even determine a non-trivial upper bound. Here we give a framework for further analysis.

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